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PRACTICAL MATHEMATICS

PART I

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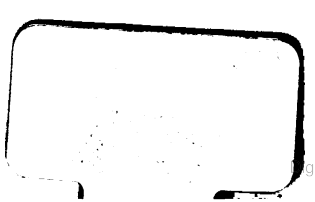

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PRACTICAL MATHEMATICS

PART I

INSTRUCTION PAPER

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PRACTICAL MATHEMATICS

PART I

INTRODUCTION

No one who is at all acquainted with the demands of the Engineering profession will deny the need of a good foundation in elementary mathematics any more than he will deny the need of a solid underpinning on which to rest the walls of a big business block.

The simplest problems of the contractor and workman, such as the number of feet of lumber required for a house, the number of cubic yards of excavation for a ditch or cellar, the proper understanding of plans and specifications, and the laying off of measurements according to these plans, all require a knowledge of this important subject. The size of a concrete retaining wall, the dimensions of a girder for a steel structure, the amount of iron in the field of a dynamo, or the capacity of the cylinders of an engine, is certainly not left to the arbitrary judgment of a foreman but is carefully worked out by mathematics and by a knowledge of the properties of the materials used.

Mathematics might be likened to a kit of tools which the workman carries; the master workman carries more than the apprentice and the more tools each man has in his kit and knows how to use, the more things he can do and the greater is his earning power. Each mathematical process is a tool to be used as the occasion demands. Some of them are used in every problem which comes up, others less frequently, but the more advanced the work the greater the number of tools required.

It is with this keen demand in mind, therefore, that we are requiring of each student at the outset of his course this work or its equivalent in Practical Mathematics. We want him to fill his kit with enough tools to meet the steady demands of the work ahead of him, and we feel sure that, once provided with this equipment, his progress will be assured.

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In the preparation of this work the authors have intentionally lost sight of the material usually found in the school books on this subject, and have kept in mind only the particular parts which are of special importance to the engineering student. Not only the topics discussed but all of the problems have been made exceptionally practical, and the aim has been at all times to give the student the satisfaction of knowing that whatever he is learning will be of use in his work and will also count for his advancement.

DEFINITIONS AND MATHEMATICAL SIGNS

1. Definitions. *Mathematics* is the science which treats of quantity, and its fundamental branches are Arithmetic, Algebra, and Geometry.

Quantity is anything which can be increased, diminished, or measured; for example: numbers, lines, space, motion, time, volume, and weight.

A *unit* is a single thing, or *one*.

A *number* is a unit or a collection of units and is either *concrete* or *abstract*.

A *concrete* number is one whose units refer to particular things, as, for example 5 rivets, 7 bolts.

An *abstract* number does not refer to any particular thing. For example, 5, 23, etc., used without designating any particular objects, are abstract numbers.

2. Mathematical Signs. For the sake of brevity, signs are used in mathematics to indicate processes. Those signs most used in Arithmetic are $+$, $-$, $\times$, $\div$, $=$, ( ) and $\overline{\quad}$

The sign $+$ is read "plus" and is the sign of addition. It shows that the quantities between which it is placed are to be added together. If 2 and 2 are to be added it is expressed, thus: $2 + 2$ are four.

The sign $-$ is read "minus" and is the sign of subtraction. It means that the quantity which follows this sign is to be subtracted or taken away from the quantity which precedes it, thus: $6 - 4$ are 2.

The sign $\times$ is read "times" and is the sign of multiplication. It means that the quantity which precedes this sign is to be multiplied by the quantity which follows it, thus: 2×5 are 10.

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The sign ÷ is read "divided by" and is the sign of division. It means that the quantity which precedes this sign is to be divided by the quantity which follows it, thus: $4 \div 2$ are 2.

The sign = is read "equals" or "is equal to" and is the sign of equality. It means that the expressions between which it is placed are identical in value, thus: $4 + 3 = 10 - 3$. This sign is very often misused. Great care should be taken at all times to make sure that the quantities connected by it are *equal.* For example, it would be absurd to say that $5 + 9 = 14 \div 2 = 7$, because $5 + 9$ does not equal 7.

The *parenthesis* ( ) and *vinculum* $\overline{\quad}$ are used to show that two or more quantities are to be treated as one; or in other words, that the operations indicated within the parenthesis or under the vinculum are to be carried out first, thus:

$$(20 - 5) + 3 - \overline{2 + 3} = (15) + 3 - (5) = 13.$$

NOTATION

3. *Notation* is the art of writing numbers in words, in figures, and in letters.

There are two methods of notation in common use; the *Roman* and the *Arabic.*

4. Roman Notation. In the *Roman notation,* 7 capital letters are used, as follows:

I	is	used	to express	one.
V	"	"	"	five.
X	"	"	"	ten.
L	"	"	"	fifty.
C	"	"	"	one hundred.
D	"	"	"	five hundred.
M	"	"	"	one thousand.

All other numbers are expressed by repetitions or by combinations of these seven letters according to the following rules:

By repeating a letter the value denoted by the letter is doubled; thus: XX means twenty; CC means two hundred.

By placing a letter denoting a less value before a letter denoting a greater, their difference of value is represented; thus: IV denotes four or one less than five; XL denotes forty or ten less than fifty.

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By placing a letter denoting a less value after a letter denoting a greater value, their sum is represented; thus: VII denotes seven or two more than five. XV denotes fifteen or five more than ten.

A line ‾ *placed over a letter increases the value denoted by the letter a thousand times;* thus: $\overline{X}$ means ten thousand. $\overline{IV}$ means four thousand.

The use of the Roman notation is now confined mainly to the writings of dates, and the numbering of chapters in books, and the hours on the dials of clocks.

Table I shows some of the combinations of the 7 letters used.

TABLE I
Roman Notation

I	one	LXXX	eighty
II	two	XC	ninety
III	three	C	one hundred
IV	four	CC	two hundred
V	five	CCC	three hundred
VI	six	D	five hundred
VII	seven	DC	six hundred
VIII	eight	DCC	seven hundred
IX	nine	CM	nine hundred
X	ten	M	one thousand
XX	twenty	MD	fifteen hundred
XXX	thirty	MM	two thousand
XL	forty	$\overline{X}$	ten thousand
L	fifty	$\overline{M}$	one million
LX	sixty	MCMX	1910
LXX	seventy		

5. Arabic Notation. The *Arabic notation* employs ten characters or figures in expressing numbers. They are

1,	2,	3,	4,	5,	6,	7,	8,	9,	0
one	two	three	four	five	six	seven	eight	nine	cipher

The first nine are sometimes called *digits;* the cipher is also called *naught* or *zero* because it expresses *nothing* or the absence of a number.

The digits (1, 2, 3, 4, 5, 6, 7, 8, 9) have been termed *significant figures* because each has of itself a definite value, always representing so many units or *ones* as its name indicates. However, the value of the units represented by a figure depends upon the particular position which that figure occupies with regard to other figures. This position is called its *place* or *order.*

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For example, if three figures are written together to represent a number, as 444, each of these figures, without regard to its place, expresses four units, but when considered as part of the number these fours differ in value. The 4 in the first place to the right represents *4 units;* the 4 in the second place, represents *4 tens* or 4 units each ten times the size or value of a unit of the first place; and the 4 in the third place, represents *4 hundreds*, or 4 units each one hundred times the size or value of a unit of the first place. It is readily seen that the value of any figure is increased ten-fold by removing it one place to the left.

hundreds	tens	units	
		4	= 4 units
	4	0	= 4 tens
4	0	0	= 4 hundreds
4	4	4	= total

The cipher becomes significant when connected with other figures by filling a place that otherwise would be vacant, as in 10 (ten) it gives a ten-fold value to the 1. In 130 (one hundred thirty) it gives a ten-fold value to the 13. A cipher between two or more figures produces the same effect. In 405 the cipher which fills the intervening place between 4 and 5 causes the 4 to represent four *hundreds*, not four tens.

hundreds	tens	units	
		5	= 5 units
	0	0	= 0 tens
4	0	0	= 4 hundreds
4	0	5	= total

The following general principles should be firmly fixed in the mind:

All numbers are expressed by the nine digits and zero.

Zero has no value; it is used to fill vacant places only.

A figure has different values according to the place it occupies.

The base of the system of notation is ten; ten units of any order making one unit of the next higher order.

NUMERATION

6. *Numeration* is the art of reading numbers when expressed by letters or figures.

This is accomplished by first enumerating the *orders* from right to left, as shown in Table II, and then reading these orders in the reverse direction in groups of three, called *periods*. The first three orders, *Units, Tens, Hundreds,* constitute the *first* or *unit period.* The second three orders form the *second* or *thousand period;* the third three orders, the *third* or *million period;* and so on.

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The system of periods is shown clearly in Table III. Such a number as 534 consists of one period and is read—five hundred thirty-four. The number shown below the orders in Table II is read—two million, seven hundred fifteen thousand, six hundred thirty-nine.

TABLE II

The Orders

7th	6th	5th	4th	3rd	2nd	1st
Millions	Hundreds of Thousands	Tens of Thousands	Thousands	Hundreds	Tens	Units
2	7	1	5	6	3	9

The number in Table III, which is 987654321987654, is divided into the periods 987, 654, 321, 987, 654, and is read—nine hundred eighty-seven trillion, six hundred fifty-four billion, three hundred twenty-one million, nine hundred eighty-seven thousand, six hundred fifty-four.

TABLE III

The Periods

Hundreds, Tens, Units of Trillions	Hundreds, Tens, Units of Billions	Hundreds, Tens, Units of Millions	Hundreds, Tens, Units of Thousands	Hundreds, Tens, Units of Units
9 8 7	6 5 4	3 2 1	9 8 7	6 5 4
5th Period	4th Period	3rd Period	2nd Period	1st Period

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PROBLEMS FOR PRACTICE

Write in words:

1. 18,765,972.
2. 834,769,780.
3. 3,576,879,421.
4. 10,805,056. Ans. Ten million eight hundred five thousand fifty-six.

Write in figures:

5. Seventy-eight million, forty-one thousand seven.
6. One thousand three. Ans. 1,003.
7. Five hundred six thousand.
8. Ninety million, two thousand, three hundred twenty-seven.
9. Three hundred five thousand seventy-nine.
10. Eight hundred sixty-four million, four thousand, twenty.

ADDITION

7. *Adding* is the process of finding a number which is equal to the combined values of two or more given numbers. The result thus obtained is called the *sum*. Hence, it may be said that the sum of two or more numbers is a number containing as many units as all the numbers taken together. Thus the sum of 5 rivets and 7 rivets is 12 rivets, since 12 contains as many units as 5 and 7 together.

Letters of the alphabet are also used to represent quantities. A letter may stand for any number, and the same letter may have a different value in different problems. For example, a may equal 3 in one problem, 7 in another, and 13 in still another. b may equal 2 in one problem, and 5 in another, 6, 10, or 15, etc. c may equal 2, 3, 9, 25, or any other number, and so on with other letters. The sum of a, b, and c will be equal to $a + b + c$, just as the sum of 2, 5, and 7 will be equal to $2 + 5 + 7$. Now, if $a = 2$, $b = 5$, and $c = 7$, the sum of a, b, and c, is found by simply adding together 2, 5, and 7, obtaining 14; on the other hand if the values of a, b, and c are not known, their sum can only be indicated. If a and b represent the horse-powers of two different engines or of two weights lying in a scale-pan, then $a + b$ equals the total horse-power or the total weight.

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8. Since a number is a collection of units of the same kind, two or more numbers may be united into one sum only when the units are of the same kind; thus: 2 bolts + 4 bolts = 6 bolts, but 4 rivets and 3 bolts cannot be united into one quantity, either of bolts or of rivets. On the other hand, if quantities of different kinds are represented by letters, they may be added together in order to help solve the problem. For example, if a represents a car, and b an engine, their sum may be indicated $(a + b)$. It must be remembered, however, that the car and the engine have not actually been added together, because they represent units of different kinds. The use of letters makes the solution of certain kinds of problems much simpler, and later will be taken up more fully.

Rules for Addition. *Write the numbers under each other, placing them so that units are under units, tens under tens, hundreds under hundreds, and so on.*

Add up the column of units; and put the right-hand figure of this sum under the unit column, carrying the remaining figure or figures to the column of tens; add up the tens column, including the carried figures, put down the right-hand figure and carry as before. Continue in this way until the last column is reached, putting down the total of the last column to give the final sum.

Examples. 1. Find the sum of 567; 141; and 93.

SOLUTION. Write these numbers under one another, so that the units of each shall be in the same vertical column. Then add up as follows: 3 and 1 are 4, and 7 are 11. Place the right-hand figure 1 under the units column and carry 1 ten to the next column. Adding the tens column, 1 (carried) + 9 are 10 and 4 are 14 and 6 are 20. Put down the right-hand figure, which is zero, and carry 2 to the next column; then 2 (carried) and 1 are 3 and 5 are 8. Putting down the 8, the total sum is found to be 801.

```
 567
 141
  93
 ---
 801
```

2. Find the sum of 6,321; 2,576; 9,702; and 257.

SOLUTION. Write these numbers one under the other as before. Add up the unit column as follows: 7 and 2 are 9, and 6 are 15, and 1 are 16. Put down the 6 and carry the 1; then adding the tens column, 1 (carried) and 5 are 6, and 7 are 13, and 2 are 15. Put down the 5 and carry the 1; then adding the hundreds column, 1 (carried) and 2 are 3, and 7 are 10, and 5 are 15, and 3 are 18; put down the 8 and carry the 1. Then adding the thousands column, 1 (carried) and 9 are 10, and 2 are 12, and 6 are 18. Putting down the whole amount, 18, gives the total sum of the numbers as 18,856.

```
 6321
 2576
 9702
  257
-----
18856
```

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3. Add the following quantities: $a + 2b + c$, $3b + 2c$, $6a + b + c$, and $10b$.

SOLUTION. Write the quantities in columns, keeping the like terms under each other; that is, keep the a's in the first vertical column, the b's in the second, the c's in the third. The sum of the a's is $a + 6a$ or $7a$. $2b + 3b + b + 10b$ are $16b$. $c + 2c + c$ are $4c$. Connecting the sums by the sign of addition the total is found to be $7a + 16b + 4c$. When the number showing how many times the letter is taken is omitted, the letter is to be taken once. Thus: $a = 1a$, $b = 1b$, $c = 1c$.

$$\begin{array}{rrr} a+ & 2b+ & c \\ & 3b+ & 2c \\ 6a+ & b+ & c \\ & 10b & \\ \hline 7a+ & 16b+ & 4c \end{array}$$

9. Proof. To prove that a sum is correct, begin at the top and add the columns *downward* in the same manner as they were added upward; if the two sums agree, the work is presumably correct, for adding downward inverts the order of the figures, and therefore any error made in the first addition would probably be detected in the second.

PROBLEMS FOR PRACTICE

Find the sum of

1. $56 + 49 + 17 + 36 + 21$. Ans. 179
2. $42 + 46 + 43 + 58 + 91$.
3. $467 + 536 + 84 + 705$. Ans. 1,792
4. $2{,}008 + 1{,}400 + 706 + 300 + 77$.
5. $8{,}950 + 15{,}765 + 7{,}732$. Ans. 32,447
6. $26{,}661 + 8{,}735 + 6{,}877 + 33{,}413$.
7. $8{,}792 + 980 + 5{,}607 + 89$.
8. $346 + 4{,}682 + 64 + 798 + 21$.
9. $26 + 425{,}902 + 3{,}006 + 490 + 36{,}221$. Ans. 465,645
10. $3a + 11b + a + 7a + 5b$. Ans. $11a + 16b$
11. $c + 2a + 5c + 4a + 3c$. Ans. $6a + 9c$
12. $6a + b + 5d + 2d$. Ans. $6a + b + 7d$

10. It has now been shown how to add numbers correctly when the process is indicated. It often happens, however, that problems will be met in which only the statement of the relations between quantities are given. The following illustrative example will show the method of solving problems of this nature.

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Example. An electric light plant, capable of furnishing current for 200 16-candle-power lamps cost as follows:

16 Horse-power engine	350	dollars
Dynamo	275	"
Driving belt	50	"
Installation	35	"

What was the total cost?

SOLUTION. In an example of this kind the problem should be read several times until it is thoroughly understood. If the numbers to be used in computation are fixed in mind the student will not be misled by the wording. For example, it can readily be seen that 200 will not be used in the computation. Find the total cost of the equipment as shown in the margin.

350
275
50
35
710

PROBLEMS FOR PRACTICE

1. A marine engine during a 3 hours' run makes 9,187 revolutions the first hour, 9,062 the second, and 9,233 the third. How many does it make in the 3 hours?

2. Coal is fed to a furnace as follows: Monday, 376 pounds; Tuesday, 307 pounds; Wednesday, 438 pounds; Thursday, 425 pounds; Friday, 399 pounds; Saturday, 301 pounds. Find the total for the week.

3. The items for lumber called for in a contract were—frame, 3,896 feet; flooring, 6,796 feet; finish, 2,739 feet. How many feet were used?

4. A surveying party works six weeks. The first week they survey 151 miles; the second week, 111 miles; the third week, 162 miles; the fourth week, 159 miles; the fifth week, 96 miles; the sixth week, 48 miles. How many miles did they survey?

5. There are five water wheels installed in a water power plant. The power furnished by the first wheel is 2,225 horse-power, and the others furnish, 3,150, 4,275, 5,650, and 8,275 horse-power. What is the total capacity of the five wheels?

6. The weekly capacity of 4 lathes is as follows: 2,500 castings, 4,175 nuts, 3,420 brass boxings, and 2,185 finished trimmings. How many pieces do the four lathes turn out per week?

7. When purchasing an 85 dollar indicator, the following extras were bought:

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1 Spring	5	dollars
1 Elbow	3	"
Pantograph	10	"
Speed Counter	2	"
Planimeter	20	"

What was the total cost of the outfit including the indicator?

8. If a 10-inch belt will transmit 17 horse-power at a speed of 1,800 feet per minute, and a 16-inch belt will transmit 36 horse-power at the same speed, how much power will be transmitted by the two belts?

SUBTRACTION

11. *Subtraction* is the process of finding the difference between two quantities; this difference when added to the smaller will give a result equal to the greater.

For example, the difference between 16 and 7 is 9, since 7 added to 9 makes 16. The greater of the two quantities whose difference is to be found, is called the *minuend;* the smaller is called the *subtrahend.* The quantity left after taking the subtrahend from the minuend is called the *difference* or *remainder.*

Only like quantities and units of the same order can be subtracted, and the remainder is always like the minuend and subtrahend.

It can readily be seen that subtraction is the reverse of addition, and this fact is made use of to prove subtraction as shown in the following:

Example. Subtract 114 from 237 and prove the result.

SOLUTION. Beginning with the units column, 4 (units) are subtracted from 7 (units) leaving 3 (units), which is set down directly under the column in units place. Proceeding to the next column 1 (ten) is subtracted from 3 (tens) leaving 2 (tens), which is set down in tens place. Proceeding as before, the final remainder is found to be 123.

	Proof
237	114
114	123
123	237

Rules for Subtraction. *Write the subtrahend under the minuend so that units of the same order will be in the same column.*

Begin with the units of the lowest order to subtract, and proceed to the highest, writing each remainder under the line in its proper place.

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If any digit of the minuend is less than the corresponding digit of the subtrahend, add ten to it and then subtract; but consider that the next digit of the minuend has been diminished by one.

Examples. 1. From 6,784 subtract 3,776.

SOLUTION. In order to subtract 6 in the units place in the subtrahend from 4 in the units place in the minuend, the 4 must first be increased by 10 giving 14; then 6 subtracted from 14 leaves 8 in the units place of the remainder. But the 10 added to the 4, has been obtained by diminishing the 8 in the tens place of the minuend by 1, leaving 7. The 7 in the tens place of the subtrahend subtracted from this leaves zero in the tens place of the remainder. The 7 in the hundreds place of the subtrahend subtracted from the 7 in the hundreds place of the minuend leaves zero in the hundreds place of the remainder, and the 3 in the thousands place of the subtrahend subtracted from the 6 in the thousands place of the minuend leaves 3 in the thousands place of the remainder, giving 3,008.

6784 minuend
3776 subtrahend
3008 remainder

2. From 1,000 subtract 621.

SOLUTION. The 0 in the units place of the minuend must first be increased by 10; then 1 subtracted from 10 leaves 9 in the units place of the remainder. In adding 10 to the 0, 1 has been taken from the tens place, but as it was itself 0, it had to borrow from the next place and continue borrowing until a numerical place was reached. Proceeding in this way the total remainder 379 is obtained.

1000 minuend
621 subtrahend
379 remainder

12. For rapid computation, the *making change method* is to be preferred. It can be readily learned and is illustrated as follows:

Example. Find the difference between 8,763 and 4,258.

SOLUTION. Think "8 + 5 = 13," and write 5; "1 (carried) + 5 + 0 = 6," and write 0; "2 + 5 = 7," and write 5; "4 + 4 = 8," and write 4. Check the result thus: 4505 + 4258 = 8763.

8763
4258
4505

This method of making change can be used to good advantage in finding the difference between a minuend and several subtrahends.

Example. A man had on deposit in his bank 350 dollars. He withdrew at various times 6 dollars, 17 dollars, and 84 dollars; find his balance in the bank. Arrange the numbers as shown in the margin.

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SOLUTION. Seventeen (4 + 7 + 6) and three are 20; write 3 and carry 2. Eleven (2 carried + 8 + 1) and four are 15; write 4 and carry 1. One and two are 3; write 2. The man's balance is 243 dollars.

350	dollars
6	"
17	"
84	"
243	"

13. Letters may be used in subtraction just as in addition. For instance, if a quantity represented by b is to be subtracted from a quantity represented by a, the difference will be represented by the quantity $(a - b)$; or if c is to be subtracted from b, the difference will be represented by the quantity $(b - c)$.

PROBLEMS FOR PRACTICE

1. From 7,282 subtract 4,815. Ans. 2,467
2. From 64,037 subtract 5,908.
3. From 6,231 subtract 3,084. Ans. 3,147
4. From 1,740,932 subtract 807,605.
5. From 71,287 subtract 40,089. Ans. 31,198
6. From 1,000,000 subtract 999,999.

Use the *making change method* in Problems 7, 8, and 9.

7. Deposit, 850 dollars; checks drawn, 105 dollars, 72 dollars, 183 dollars. Find balance.

8. Deposit, 625 dollars; checks drawn, 130 dollars, 210 dollars, 128 dollars. Find balance.

9. Deposit, 475 dollars; checks drawn, 78 dollars, 162 dollars, 24 dollars. Find balance.

10. From $5a + 2b$ subtract $3a + b$. Ans. $2a + b$

11. From $a + 5b + 2d$ subtract $a + 4b + 2d$. Ans. b

12. From a tank containing 935 gallons of water, 648 gallons were drawn off. Then 247 gallons ran in. How many gallons were then in the tank? (Suggestion: Subtract 648 from 935 and add 247.)

13. A man purchased 8,983 bricks, but used only 5,363. How many had he left?

14. A coal shed contains 8,579 tons. 3,243 tons are taken from it. It then receives 4,112 tons more. After that 1,602 tons are taken out of it. How many tons remain?

15. An electric power plant can generate 2,000 horse-power. Of this, 1,910 horse-power is used. The manager then agrees to

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furnish another firm with 784 horse-power. How much more power will he need? (Suggestion: Add 784 to 1,910 and subtract 2,000.)

Ans. 694 horse-power.

16. An engine develops 147 horse-power. 16 horse-power is used in running the engine itself. How much power is available for running machinery?

17. 1,200 gallons are pumped from a tank. Of this, 32 gallons are lost in leakage, etc. How much is discharged by the pump?

18. A 75 horse-power boiler evaporates 2,140 pounds of water into steam per hour. One engine uses 1,310 pounds, another uses 417 pounds, and a pump requires the remainder. How much steam is used by the pump? (Suggestion: Add 417 to 1,310 and subtract from 2,140.) Ans. 413 pounds.

19. An engine makes 54,000 revolutions in a day of 12 hours. A motor makes 720,000 revolutions in the same time. By how many revolutions per day does the speed of the motor exceed that of the engine?

20. It takes 3,880 pounds of steam per hour to run a certain 160 horse-power engine. If it takes 1,940 pounds to run a 60 horse-power engine and 2,140 pounds to run a 72 horse-power engine, does the largest engine require as much steam as the two small ones? (Suggestion: Add 1,940 to 2,140 and compare with 3,880.)

MULTIPLICATION

14. *Multiplication* is a short method of adding a quantity to itself a certain number of times; or, it is the process of taking one quantity as many times as there are units in another.

It is known that $2 + 2 + 2 + 2 + 2 = 10$; but this same process may be expressed more briefly by the aid of multiplication, thus: $5 \times 2 = 10$. The 5 shows how many twos are used in adding. This last expression is read, "five times two equals ten."

In multiplication three terms are employed—the multiplicand, the multiplier, and the product.

The *multiplicand* is the quantity to be multiplied or taken.

The *multiplier* denotes the number of times the multiplicand is to be taken.

The *product* is the result or quantity obtained by the multiplication.

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Another term, called the *coefficient*, is used to indicate the numerical part of a quantity which consists of numbers and letters multiplied together. Thus in the expression $3ab$, 3 is the coefficient of ab, and denotes that ab is taken 3 times. Sometimes it is convenient to consider any part of the product as the coefficient of the remaining part. Thus in $3ab$, $3a$ might be considered the coefficient of b, or $3b$ the coefficient of a. A coefficient is, therefore, called *numerical* or *literal* according as it is a number or one or more letters. When no numerical coefficient is expressed, 1 is always understood,

TABLE IV
Multiplication Table

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25
2	4	6	8	10	12	14	16	18	20	22	24	26	28	30	32	34	36	38	40	42	44	46	48	50
3	6	9	12	15	18	21	24	27	30	33	36	39	42	45	48	51	54	57	60	63	66	69	72	75
4	8	12	16	20	24	28	32	36	40	44	48	52	56	60	64	68	72	76	80	84	88	92	96	100
5	10	15	20	25	30	35	40	45	50	55	60	65	70	75	80	85	90	95	100	105	110	115	120	125
6	12	18	24	30	36	42	48	54	60	66	72	78	84	90	96	102	108	114	120	126	132	138	144	150
7	14	21	28	35	42	49	56	63	70	77	84	91	98	105	112	119	126	133	140	147	154	161	168	175
8	16	24	32	40	48	56	64	72	80	88	96	104	112	120	128	136	144	152	160	168	176	184	192	200
9	18	27	36	45	54	63	72	81	90	99	108	117	126	135	144	153	162	171	180	189	198	207	216	225
10	20	30	40	50	60	70	80	90	100	110	120	130	140	150	160	170	180	190	200	210	220	230	240	250
11	22	33	44	55	66	77	88	99	110	121	132	143	154	165	176	187	198	209	220	231	242	253	264	275
12	24	36	48	60	72	84	96	108	120	132	144	156	168	180	192	204	216	228	240	252	264	276	288	300
13	26	39	52	65	78	91	104	117	130	143	156	169	182	195	208	221	234	247	260	273	286	299	312	325
14	28	42	56	70	84	98	112	126	140	154	168	182	196	210	224	238	252	266	280	294	308	322	336	350
15	30	45	60	75	90	105	120	135	150	165	180	195	210	225	240	255	270	285	300	315	330	345	360	375
16	32	48	64	80	96	112	128	144	160	176	192	208	224	240	256	272	288	304	320	336	352	368	384	400
17	34	51	68	85	102	119	136	153	170	187	204	221	238	255	272	289	306	323	340	357	374	391	408	425
18	36	54	72	90	108	126	144	162	180	198	216	234	252	270	288	306	324	342	360	378	396	414	432	450
19	38	57	76	95	114	133	152	171	190	209	228	247	266	285	304	323	342	361	380	399	418	437	456	475
20	40	60	80	100	120	140	160	180	200	220	240	260	280	300	320	340	360	380	400	420	440	460	480	500
21	42	63	84	105	126	147	168	189	210	231	252	273	294	315	336	357	378	399	420	441	462	483	504	525
22	44	66	88	110	132	154	176	198	220	242	264	286	308	330	352	374	396	418	440	462	484	506	528	550
23	46	69	92	115	138	161	184	207	230	253	276	299	322	345	358	391	414	437	460	483	506	529	552	575
24	48	72	96	120	144	168	192	216	240	264	288	312	336	360	384	408	432	456	480	504	528	552	576	600
25	50	75	100	125	150	175	200	225	250	275	300	325	350	375	400	425	450	475	500	525	550	575	600	625
1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

as a is the same as $1a$. When a coefficient occurs just before a parenthesis, it indicates that every term within the parenthesis is to be multiplied by that coefficient.

To multiply with accuracy and rapidity, the product of any two quantities, at least from 2 to 12, must be known at sight. The combinations of these should be practiced until they can be given correctly and without hesitation.

The following points should also be fixed firmly in mind: Zero times any quantity or any quantity times zero is zero. For example,

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$0 \times 0 = 0$; $0 \times 8 = 0$; $942 \times 0 = 0$, $b \times 0 = 0$, etc. One times any quantity or any quantity times one gives that quantity as a product. For example, $1 \times 7 = 7$, $85 \times 1 = 85$, $1 \times 1 = 1$, $a \times 1 = a$.

Table IV gives the product of any two numbers up to 25×25. To use this table, find one of the two numbers in the upper row and the other in the left-hand column. The product of the two will be found at the intersection of two imaginary lines drawn parallel to the heavy lines shown in the table at 12×12 and 20×20. For example under the 9 and opposite the 8 is found the product, 72. That is, $9 \times 8 = 72$.

15. Rules for Multiplication. *The multiplicand may be either concrete or abstract. The multiplier is always abstract. The product is always like the multiplicand.*

When both quantities are abstract, either may be considered as the multiplicand or the multiplier, for the result is the same; thus: 5 times 2 is the same as 2 times 5.

Each figure of the multiplicand is multiplied by each significant figure of the multiplier, and the right-hand figure of each product is placed under the figure of the multiplier used to obtain it. The sum of the several products will be the entire product. When there is a zero in the multiplier, multiply by the significant figures only, taking care to place the right-hand figure of each separate product under the figure used in obtaining it.

Examples. 1.

SOLUTION. Having written the multiplier under the unit of the multiplicand, multiply the 5 units by 7, obtaining 35. Then set down the 5 units directly under the 7 and carry the 3; in other words, reserve the 3 tens for the tens column. Next multiply the seven tens by 7, obtaining 49, and add the 3 which is carried, and obtain 52 tens (which is the same as 5 hundreds and 2 tens). Set down 2 tens and carry the 5 hundreds; multiply 1 and 7 and add the 5 which was carried, making 12, which can be written down in full.
The product then reads, 1,225.

175 multiplicand
7 multiplier
1225 product

2. Find the product of 145 and 13.

SOLUTION *a*. It can be seen that $13 = 10 + 3$, hence the product of 13×145 will be the same as $(10 \times 145) + (3 \times 145)$. Adding the products gives 1,885 as a result.

$145 \times 3 = 435$
$145 \times 10 = 1450$
1885

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SOLUTION *b*. The same result is obtained, however, if the numbers are arranged as follows:

Commence with 3, multiply through and write the product 435. Under this write the product 1,450 obtained by multiplying by 10. In this latter product the 0 may be discarded but it must be remembered to write the 5 under second place. Adding these two products, called *partial products*, gives the final product 1,885.

```
 145
  13
 435
145
1885
```

3. Multiply 1246 by 235.

SOLUTION. Note that the three partial products are the results of multiplying by 5, 3, and 2 where each successive partial product is set one place further to the left than the preceding one. Note that 0 is under 5, 8 is under 3, and 2 is under 2; in other words, the first figure of each partial product is placed under the digit used to obtain it.

```
 1246
  235
 6230
 3738
2492
292810
```

Two special cases not covered by the general rules given above should be here considered.

1. When the digits of the multiplier are separated by ciphers:

Example. Multiply 13,456 by 2,004.

SOLUTION. Although the multiplier contains four digits, in the short method only two partial products appear.

The first figures obtained by multiplying by 4 and 2 appear under these respective digits, the zeros simply marking the absence of any other characters in the product.

Regular method	Short method
13456	13456
2004	2004
53824	53824
00000	26912
00000	26965824
26912	
26965824	

2. When ciphers are at the right of multiplier or multiplicand:

Example. Multiply 5,760 by 3,000.

SOLUTION. In this case it is necessary only to multiply 576 by 3 giving 1,728; then annex the total number of ciphers found at the right of both multiplier and multiplicand, in this case four, giving as the final result, 17,280,000.

```
 5760
 3000
17280000
```

16. Short Methods. *To multiply by ten and *powers of ten.* This method is only a slight variation of that given in the previous paragraph.

* A power of ten is the product obtained by multiplying ten by itself a given number of times. Thus, 1,000 and 10,000 are respectively the third and fourth powers of 10.

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Annexing a cipher to a whole number multiplies that number by ten, etc. Thus, to multiply 378 by 1,000 write at once 378,000.

To multiply by a number a little less than 10, 100, or 1000 the process may be shortened as shown in the following:

Examples. 1. Multiply 254 by 99.

SOLUTION. 99 is 100 diminished by 1; hence, multiply 254 first by 100 and then by 1 and subtract the results.

$$\begin{array}{r} 25400 \\ 254 \\ \hline 25146 \end{array}$$

2. Multiply 196 by 997.

SOLUTION. 997 is 1,000 diminished by 3, hence, multiply 196 first by 1,000 and then by 3 and subtract the results.

$$\begin{array}{r} 196000 \\ 588 \\ \hline 195412 \end{array}$$

To multiply by 25 annex two ciphers to the multiplicand and divide by 4.

To multiply by 11:

(*a*) *When the multiplicand contains two figures,* place their sum between them. If this sum is greater than 10, carry 1 to the third place.

Example. Multiply 47 by 11.

SOLUTION. Place 7 in units place. Add 4 and 7. Putting 1 in tens place and carrying 1. Place (4 + 1) in hundreds place. Result 517.

(*b*) *When the multiplicand is any number,* write the right-hand figure in units place; then add the first and second, second and third, and so on, finally setting down the left-hand figure. Carry as usual.

Example. Multiply 365 by 11.

SOLUTION. Write 5; 5 + 6 = 11; write 1 and carry 1; 1 carried + 6 + 3 = 10; write 0; 1 carried + 3 = 4; write 4, making the result, 4,015.

$$\begin{array}{r} 365 \\ 11 \\ \hline 4015 \end{array}$$

17. A Method of Checking Multiplication. *Add separately the figures of the multiplicand and multiplier until they are reduced to one figure each; then multiply these together and again reduce this product by addition to one figure. If the multiplication is correct, the final result should check with the successive additions of the figures of the product.*

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Example. Multiply 6,547 by 301 and check.

SOLUTION. The solution is shown in the margin. To check proceed as follows: Taking the multiplicand, $6 + 5 + 4 + 7 = 22$; $2 + 2 = 4$. Again for the multiplier, $3 + 1 = 4$. Their product, $4 \times 4 = 16$; $1 + 6 = 7$.

Treating the product in the same manner; $1 + 9 + 7 + 6 + 4 + 7 = 34$; $3 + 4 = 7$. The multiplication is correct.

$$\begin{array}{rr} 6547 & -\ 4 \\ 301 & -\ 4 \\ \hline 6547 & 16 - 7 \\ 19641 & \\ \hline 1970647 & -\ 7 \end{array}$$

PROBLEMS FOR PRACTICE

18. Letters may be used in multiplication as follows: The product of a and b is $a \times b$, and the product of a, b, and c is $a \times b \times c$. If $a = 6$, $b = 4$, and $c = 1$, the product of a, b, and c is $6 \times 4 \times 1$ or 24. The common practice is to omit the multiplication sign when letters are multiplied by letters or numbers thus: $2 \times x$ is written $2x$; $2a \times 3b$ is written $6ab$; $3a \times 4b \times 5c$ is written $60abc$.

Multiply:

1. 2,928 by 364. Ans. 1,065,792
2. 7,319 by 394.
3. 5,698 by 792. Ans. 4,512,816
4. $9a$ by $2b$. Ans. $18ab$
5. $3ab$ by $4cd$. Ans. $12abcd$
6. 3,186 by 839. Ans. 2,673,054
7. 42,308 by 692.
8. 876 by itself.
9. 78 by 10.
10. 57 by 1,000.
11. 52 by 99.
12. 16 by 25.
13. 92 by 11.
14. 103 by 25.

15. There are 746 watts in a horse-power. How many watts are there in 20 horse-power? (Suggestion: Multiply 746 by 20.)

16. A piston has an area of approximately 113 square inches. If the steam pressure is 47 pounds per square inch, what is the total pressure upon it? Ans. 5,311 pounds.

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17. The head of a boiler has an area of 11,310 square inches. If the pressure per square inch is 40 pounds, what is the total pressure on the head? Ans. 452,400 pounds.

18. A concrete mixer delivers 14 cubic yards per hour. A gang of men takes away 12 cubic yards per hour. How many cubic yards will remain unmoved at the end of 4 hours? At the end of 8 hours?

19. If there are 12 threads per inch on a screw, how many threads are there in 4 inches?

20. If a piston moves through 468 feet in one minute, how far does it travel in 45 minutes? Ans. 21,060 feet.

21. If a boiler evaporates 1,945 pounds of water in one hour, how many pounds will it evaporate in 9 hours?

22. A certain girder supports 136,925 pounds. How much will 65 such girders support? Ans. 8,900,125 pounds.

23. An engine in a certain power plant requires 18 pounds of steam per horse-power per hour. If the engine is developing 640 horse-power, what is the total steam consumption?

24. A gang of men can lay 1 mile of track in 5 days. How long will it require them to lay 35 miles?

25. If a pump discharges 4 gallons of water per stroke and it makes 100 strokes per minute, how many gallons will it discharge in 45 minutes? Ans. 18,000 gallons.

26. If one 32 candle-power incandescent lamp costs one cent per hour, what will be the cost of 48 such lamps for 13 hours?

27. A ton of a certain kind of fuel occupies 42 cubic feet. What space will 73 tons occupy?

28. An engine makes 340 revolutions per minute. How many revolutions will it make in 8 hours? (Suggestion: 8 hours = 480 minutes.)

DIVISION

19. *Division* is a process of finding how many times one quantity contains another. In division there are three principal terms, the dividend, the divisor, and the quotient or answer.

The *dividend* is the quantity to be divided.

The *divisor* is the quantity which is divided into the dividend.

The *quotient* is the number of times the divisor is contained in the dividend.

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When the dividend does not contain the divisor an exact number of times, the excess is called the *remainder.* The remainder being a part of the dividend will always be of the same kind as the dividend and must necessarily be less than the divisor.

Division may be indicated in any of the following ways: $24 \div 2$; $\frac{24}{2}$; $2\overline{)24}$

Division is the reverse of multiplication, as shown by the following:

$6 \times 7 = 42$	$42 \div 6 = 7$	$42 \div 7 = 6$
$5 \times 8 = 40$	$40 \div 8 = 5$	$40 \div 5 = 8$

20. There are two distinct methods used, viz, *long division* and *short division;* in the former all the work is written out but in the latter the process is performed mentally and the result only is written. Short division is generally used when the divisor does not exceed 12.

The following examples illustrate the two processes.

Examples. 1. Divide 720 by 5.

Long Division

144 quotient

5)720

5

22

20

20

20

Short Division

5)720

144 quotient

SOLUTION. In long division it is found that 5 is contained in 7 once. Write 1 as the first figure of the quotient, and subtract, giving a remainder of 2. To the remainder *annex the next figure of the dividend, and divide as before, obtaining 4 as the second figure of the quotient. Annex 0 which is the next figure of the dividend, and divide again by 5, obtaining 4 as the last figure of the quotient with no remainder. The division is now complete.

It often happens, after bringing down a figure from the dividend, that the number is too small to contain the divisor. In this case place a zero in the quotient, and continue bringing down the figures from the dividend until the number thus formed will contain the divisor.

* In the above solution the term "annex," meaning *to place after,* is used. The opposite term "prefix," meaning *to place before,* is also often used. Be careful to make a distinction between "annex" and "add."

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2. Divide 10,413 by 13.

SOLUTION.

$$
\begin{array}{r}
801 \\
13\overline{)10413} \\
104 \\
\hline
13 \\
13 \\
\hline
\end{array}
$$

21. Rules for Long Division. *Write the divisor and dividend in the order named, and draw a curved line between them.*

Find how many times the divisor is contained in the left-hand figure or figures of the dividend, and write the number in the quotient over the dividend.

Multiply the divisor by this figure of the quotient, writing the product under that part of the dividend from which it was obtained; subtract, and to the remainder annex the next figure of the dividend.

Find how many times the divisor is contained in the number thus formed, and write the figure denoting it at the right hand of the last figure of the quotient.

Proceed in this manner until all the figures of the dividend are divided. If there is a remainder after dividing all the figures of the dividend, place the remainder over the divisor with a line between them, and annex to the quotient.

Example. Divide 5,441 by 26.

SOLUTION.

$$
\begin{array}{r}
209\frac{7}{26} \\
26\overline{)5441} \\
52 \\
\hline
241 \\
234 \\
\hline
7 \text{ remainder}
\end{array}
$$

The proper remainder is, in all cases, less than the divisor. If, in the course of the operation, it is found to be larger than the divisor, this indicates that there is an error in the work and that the figure in the quotient should be increased.

In division, the quotient is always abstract, since it shows how many times the divisor is contained in the dividend.

Proof. In order to prove division, multiply the quotient by the divisor, and add the remainder, if there is any. If the quantity thus obtained gives the dividend, the work is correct.

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22. *Short Methods. *To divide by ten and powers of ten.* From the right in the dividend point off as many places as the divisor contains ciphers. The figures so cut off represent the remainder, to be written over the divisor and annexed to the quotient.

Find the complete quotients of the following: $530 \div 10$; $6{,}562 \div 100$; $28{,}563 \div 1{,}000$.

To divide by multiples of ten.

Example. Find the quotient of $18{,}653 \div 3{,}000$.

SOLUTION. Mark off as many figures in the dividend as there are ciphers in the divisor, thus dividing by 1,000. This leaves 18 to the left of the mark to be divided by 3, giving a quotient of 6 with a remainder of 653. Place this remainder over the divisor and annex it to the 6, giving, finally, $6\frac{653}{3000}$.

$$6\ \frac{653}{3000}$$

$$3000\overline{)18|653}$$

$$\underline{18}$$

$$653$$

To divide by 25.

Multiply the dividend by 4 and divide the product by 100, by cutting off two figures to the right.

23. Letters may be used in division as follows: a divided by b may be expressed as $a \div b$ or $\frac{a}{b}$; a divided by c, as $a \div c$, or $\frac{a}{c}$. If $a = 6$ and $b = 3$, a divided by b will equal $6 \div 3 = 2$. If $a = 6$ and $c = 2$, a divided by c will equal $6 \div 2$ or 3.

PROBLEMS FOR PRACTICE

Divide:

1. 65,814 by 6. Ans. 10,969
2. $6ab$ by $3b$. Ans. $2a$
3. 3,870 by 10. Ans. 387
4. 24 $abcd$ by $6d$. Ans. $4abc$
5. 5,321 by 10.
6. 9,473 by 100.
7. 13,987 by 1000.
8. 563 by 25.
9. If coal costs 5 dollars per ton, how many tons may be bought for 275 dollars? (Suggestion: Divide 275 by 5.)

* These methods may be made more comprehensive and valuable after the subject of Decimals has been discussed.

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10. A steamer runs 276 miles in 23 hours. What is her average speed per hour? (Suggestion: Divide 276 by 23.)

11. How many 30-foot rails are there in a double track (four rails) two miles long? There are 5,280 feet in one mile. (Suggestion: Divide $2 \times 4 \times 5{,}280$ by 30.)

12. If 9 days' work will pay for 6 tons of coal at 6 dollars per ton, what is the price of a day's labor?

13. A company furnishes equal power to 26 establishments. The total horse-power is 8,450. How much does each receive?

14. If a locomotive goes to shop for inspection after every average run of 283 miles, how many times would it be in shop during runs aggregating 10,188 miles? Ans. 36 times.

15. A certain boiler supplies steam for heating. If there are 180 square feet of heating surface in the boiler and each radiator requires 12 square feet of heating surface of the boiler, how many radiators can be supplied by the boiler? Ans. 15 radiators.

16. If 800 cubic feet of air are required for each person, how many people can occupy a room that contains 21,600 cubic feet?

17. If 1 foot of 1 inch pipe is allowed for every 90 cubic feet of space in heating a factory, how many feet of the same pipe will be required to heat 225,000 cubic feet of space?

FACTORING

24. An *integer* is a whole number, as 1, 8, 15.

All numbers are either odd or even. An *odd* number is a number that cannot be divided by 2 without a remainder, as 3, 9, 13, etc. An *even* number is one that can be divided by 2 without a remainder, as 4, 6, 8, etc.

A *prime* number is a number which can be exactly divided only by itself or 1, as 1, 3, 5, 7, etc.

A *factor* of an integer is a number which is contained an exact number of times in the given integer.

A *prime factor* is a factor which is a prime number.

Numbers are prime to each other when they have no factor in common. Thus 7 and 11 are prime to each other; also 18 and 25, etc.

In speaking of the factors of a number it is not customary to include the number itself or 1. For this reason a prime number is said to have no factors.

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Example. What are the prime factors of 16?

SOLUTION. Divide 16 by 2 which is the least prime number greater than 1, and obtain 8 as the quotient. Since 8 may again be divided by 2, the division is carried out giving 4 as the next quotient. Next divide 4 by 2 giving a quotient of 2, which is a prime number and cannot be further divided. Now the several divisors and the last quotient, all of which are prime, are the prime factors of 16.

$$\begin{array}{r} 2)16 \\ 2)\ 8 \\ 2)\ 4 \\ \hline 2 \end{array}$$

The rule for factoring may be expressed as follows:

Divide the given number by the least prime number (greater than 1) that will divide it; and divide each quotient, in the same manner. Continue dividing until a prime number is obtained as the quotient. The several divisors and the last quotient will be the prime factors desired.

Example. What are the prime factors of 78?

SOLUTION.

$$\begin{array}{r} 2)78 \\ 3)39 \\ \hline 13 \end{array}$$ Ans. 2, 3, and 13

25. Factoring by Observation. Certain numbers have such characteristics that some of their factors may be found without the trouble of an actual operation in division. The simplest of these rules for factoring by observation are as follows:

2 is a factor of all even numbers.

3 is a factor of all numbers the sum of whose digits is divisible by 3.

4 is a factor of all numbers whose last two places are divisible by 4.

5 is a factor of all numbers ending in 5 or 0.

10 is a factor of all numbers whose last figure is 0.

Examples. 1. Factor the number 56.

SOLUTION.

$$\begin{array}{r} 2)56 \\ 2)28 \\ 2)14 \\ \hline 7 \end{array}$$ Ans. 2, 2, 2, and 7

2. Factor the number 720.

SOLUTION.

$$\begin{array}{r} 10)720 \\ 2)\ 72 \\ 2)\ 36 \\ 2)\ 18 \\ 3)\ 9 \\ \hline 3 \end{array}$$ Ans. 10, 2, 2, 2, 3, and 3

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3. Factor the number 426.

SOLUTION.

$$\begin{array}{r} 2)426 \\ 3)213 \\ \hline 71 \end{array}$$

Ans. 2, 3, and 71

Factor the following:

1. 36 — Ans. 2, 2, 3, and 3
2. 30
3. 68
4. 148
5. 189 — Ans. 3, 3, 3, and 7
6. 1,264
7. 2,552
8. 1,932
9. 91
10. 2,508 — Ans. 2, 2, 3, 11, 19

CANCELLATION

26. *Cancellation.* When a series of multiplied factors is to be divided by a second series the operation may be shortened by the process of cancellation as follows:

Examples. 1. $\dfrac{6 \times 8 \times 12 \times 18 \times 24}{2 \times 3 \times 2 \times 4 \times 6} = ?$

SOLUTION. To cancel, select any two numbers one above and one below the line, and find some number that will divide each of them without a remainder. For instance, it is seen that 2 will be contained an exact number of times in both 6 and 2. Then perform the division, crossing out both numbers and placing the results directly over and below, the numbers crossed out. Proceed in this manner until there is no longer a number below the line that can be cancelled with one above the line. Then multiply together all the numbers above the line and use this as a dividend; and multiply together those below the line, and use as a divisor.

$$\frac{\overset{3}{\cancel{6}} \times \overset{4}{\cancel{8}} \times \overset{4}{\cancel{12}} \times \overset{3}{\cancel{18}} \times \overset{6}{\cancel{24}}}{\underset{1}{\cancel{2}} \times \underset{1}{\cancel{3}} \times \underset{1}{\cancel{2}} \times \underset{1}{\cancel{4}} \times \underset{1}{\cancel{6}}} = 864$$

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2. $\dfrac{16 \times 2 \times 15 \times 4}{32 \times 6 \times 8 \times 22} = ?$

SOLUTION. In an example where the product above the line, after cancelling, is less than the product below the line, the result is allowed to stand as obtained, thus: Since 5 and 22 do not cancel into any of the other numbers the result is the product of the quantities in the numerator divided by the product of the quantities in the denominator.

$$\frac{\overset{1}{\cancel{16}} \times \overset{1}{\cancel{2}} \times \overset{5}{\cancel{15}} \times \overset{1}{\cancel{4}}}{\underset{2}{\cancel{32}} \times \underset{1}{\cancel{6}} \times \underset{2}{\cancel{8}} \times 22} = \frac{5}{88}$$

Rules. *Cancel the common factors from both dividend and divisor. Next, divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor.*

It is seen, therefore, that cancellation is merely a combination of the processes of factoring and division.

PROBLEMS FOR PRACTICE

Divide:

1. $2 \times 3 \times 8 \times 12 \times 24$ by $6 \times 4 \times 36 \times 4$. Ans. 4
2. $18 \times 24 \times 32 \times 36$ by $9 \times 48 \times 4 \times 18$.
3. $15 \times 20 \times 25 \times 27$ by $15 \times 18 \times 25 \times 10$. Ans. 3
4. $40 \times 48 \times 54 \times 60$ by $30 \times 24 \times 72 \times 3$.
5. $12 \times 60 \times 36 \times 70$ by $28 \times 5 \times 48 \times 6$. Ans. 45
6. $32 \times 36 \times 33 \times 45$ by $24 \times 30 \times 44 \times 9$.
7. 18 piers, each consisting of 5 piles, were set by 10 men in 3 days. What is the cost of driving each pile if each of the men receive 3 dollars a day?
8. A bridge transmits to 3 girders each supported by 5 columns the load of 2,250 people, 45 horses, 69 vehicles. What is the load on each column, the average weights for the people, horses, and vehicles being respectively 150 pounds, 1,100 pounds, and 1,015 pounds?
9. Each of 40 teamsters hauls 9 yards of sand per day for 4 days at 1 dollar per yard. How many men with wheelbarrows will earn the same amount in two days wheeling 3 yards per day at 1 dollar per yard? Ans. 240 men.

REVIEW PROBLEMS

1. A steam plant has two engines of 934 horse-power each. The starboard engine of a steamship develops 3,218 horse-power

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and the port engine 3,232 horse-power. How much greater power is developed in the ship than in the steam plant? Ans. 4,582 H. P.

2. A pump delivers 33,720 cubic feet of water in two hours (60 minutes each). How many cubic feet are delivered per minute? Ans. 281 cu. ft.

3. A nine-foot blower makes 175 revolutions per minute. How many revolutions will it make in 29 minutes? Ans. 5,075 rev.

4. There are 360 rivets in a hundred pounds. How many pounds will be required to afford 64,800 rivets. Ans. 18,000 pounds.

5. The heating surface in a locomotive is 88 square feet in the fire box and 792 square feet in the tubes. What is the total heating surface? How many times as much surface is there in the tubes than in the fire box? Ans. { 880 square feet / 9 times

6. The weight of a battery of eight marine boilers and their equipments is as follows:

Boilers complete with mountings	295 tons
Water in boilers	73 "
Funnels	50 "
Stoke-hole plates, floors, etc.	23 "
Feed pumps	7 "
Fans and fan-engines	8 "
Feed regulators	2 "
Tools and fittings	2 "
Spare gear	10 "

What is the weight of two such batteries?

7. At 100 degrees Fahrenheit a cubic foot of water weighs 62 pounds. At 205 degrees a cubic foot weighs 60 pounds. What is the difference in weight between 173 cubic feet of water at 100 degrees and 189 cubic feet at 205 degrees? Ans. 614 pounds.

8. A roof is composed of 11 frames. The weight of one frame in detail is as follows:

2 rafters each weighing	875 lbs.
5 rods, each weighing	176 "
16 bolts, each weighing	5 "
8 bridle-straps, each weighing	15 "
2 piers supporting rafters at ridge, average each	11 "

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6 pieces at foot of struts, average each..... 11 lbs.
4 pieces uniting rafters at junction in strut, with bolts and nuts, each..... 44 "
2 rafter shoes, each.....144 "
2 cast-iron struts, each.....154 "

What is the total weight of the roof? Ans. 40,590 pounds.

9. A double mine ventilating fan runs at the rate of 84 revolutions per minute. It gives 2,818 cubic feet of air per revolution. How many cubic feet should it give per minute?

Ans. 236,712 cubic feet.

10. If oak-tanned leather belting costs 2 dollars per foot, and four-ply stitched canvas belting costs 1 dollar per foot, what is the difference in the cost of 40 feet? Ans. 40 dollars.

11. If there are 9,326 heat units in a pound of lignite coal, how many heat units in 287 pounds?

12. If 23 square feet of No. 30 sheet iron weighs 11 pounds, how much will 23 square feet weigh if three times as thick?

13. The weight of the masonry of a bridge and an engine passing over it is 1,698,575 pounds. The engine weighs 198,560 pounds. What is the weight of the masonry?

14. A pound of Pennsylvania petroleum will theoretically evaporate about 22 pounds of water. How many pounds are necessary to evaporate 4,378 pounds of water?

15. A cubic foot of hemlock weighs 25 pounds. A cubic foot of iron weighs 450 pounds. Find the difference between the weight of 230 cubic feet of hemlock and 87 cubic feet of iron.

16. Three guy ropes are fastened to a stake. The pull on one rope is 560,800 pounds; the pull on the second is 118,421 pounds; and on the third is 104,863 pounds. What is the total pull?

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EXAMINATION PAPER

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PRACTICAL MATHEMATICS

PART I

Read Carefully: Place your name and full address at the head of the paper. Any cheap, light paper like the sample previously sent you may be used. Do not crowd your work, but arrange it neatly and legibly. *Do not copy the answers from the Instruction Paper; use your own words, so that we may be sure that you understand the subject.*

1. Write in words: 734,507,001

2. Write in figures: five hundred six million, seven thousand, eighty-three.

3. A nine-foot blower makes 175 revolutions per minute. How many revolutions will it make in 29 minutes?

4. Divide the product of $2a$ and $3b$ by $6b$.

5. Suppose a cubic foot of anthracite coal weighs 55 pounds, how many cubic feet will there be in 1,100 pounds?

6. If a equals 15, b equals 3, and c equals 12, subtract c from the sum of a and b and give the numerical result.

7. What will be the cost of building a line of telegraph 274 miles long at 967 dollars per mile?

8. If it requires 324 tons of iron rail for 3 miles of track, how many tons will be required for 476 miles?

9. A mine ventilating fan runs at the rate of 84 revolutions per minute. It gives 2,818 cubic feet of air per revolution. How many cubic feet does it give per minute?

10. A certain boiler test showed the following amounts of coal used:

Monday	720	pounds
Tuesday	712	"
Wednesday	718	"
Thursday	722	"
Friday	714	"
Saturday	704	"

If one pound of coal evaporates 8 pounds of water, how much water will be converted into steam during the week?

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11. A stone crusher is capable of yielding 26 cubic feet of broken stone per hour. If a cubic foot of stone weighs on the average 167 pounds, what weight of stone will be crushed in 4 hours?

12. To produce a long ton (2,240 pounds) of pig iron the following weights of fuel were consumed:

Charcoal	2,209	pounds
Anthracite	2,964	"
Coke	2,421	"
Raw coal	3,606	"

What amount of fuel is required to produce a pound of pig iron?

13. A 12-inch gas main delivers 1,659,360 cubic feet in 30 hours. How many cubic feet does it deliver in 1 hour?

14. The weight in detail of a steam hammer is as follows:

Tup with piston and rod	40,000	pounds
Cylinder	11,000	"
Entablature	15,000	"
Leg of hammer with guides	125,000	"
Wrought iron cross-stays	12,000	"
Foundation-plate	45,000	"
Valves, valve-gear, etc.	16,500	"
Anvil and anvil-block	37,500	"

What is the weight of 116 hammers?

15. Two internally fired tubular boilers with return flues beneath the shells have each the following heating surfaces:

Heating surface in each of 80 tubes	7	sq. ft.
" " " furnaces	115	" "
" " " combustion chamber	158	" "
" " " external shell	173	" "

What is the total heating surface of both boilers?

16. The heating surface in a locomotive is 88 square feet in the fire box and 792 square feet in the tubes. What is the total heating surface? How many times as much surface is there in the tubes as there is in the fire box?

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17. A steam plant has two engines of 934 horse-power each. One engine of a steamship develops 3,218 horse-power and the other engine 3,232 horse-power. How much greater power is developed in the ship than in the steam plant?

18. If it costs a railroad 3 dollars per passenger for transportation, how many dollars profit does the road make by carrying 487,600 passengers at 5 dollars each?

19. A locomotive tank leaks at the rate of 7 gallons per hour. If the locomotive runs at the rate of 27 miles per hour, how many gallons are lost in running 432 miles?

20. A building is 356 feet high. A workman is raised to the top and then is lowered to within 78 feet of the ground. How many feet has the man been moved? How far is he from the top?

21. One automobile has 8 cylinders costing 15 dollars each, another has 6 cylinders costing 31 dollars each. Which is the more expensive? How much?

22. The various costs of a modern dwelling were as follows:

Lot	1,800	dollars
Foundation	255	"
Frame	967	"
Plastering	128	"
Finish	1,268	"
Gas Fixtures	46	"
Electric Fixtures	38	"
Plumbing	192	"
Decorating	67	"
Labor	2,261	"

What was the total cost?

23. A canal commission in digging a canal finds the following cost per mile: Surveys, 185 dollars; right of way, 628 dollars; excavating, 36,875 dollars; locks, 2,500 dollars, bridges, 4,560 dollars. The yearly cost of upkeep per mile is 35 dollars. What is the total cost of 1 mile in 4 years?

24. A party surveys several fields. The areas are 623 acres, 547 acres, 376 acres, 117 acres, 63 acres, 19 acres. What is the total number of acres? What is the area of the two largest pieces? What is the difference between the largest and the smallest?

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25. An architect superintends the construction of four buildings, receiving 3,209 dollars, 2,562 dollars, 1,154 dollars, and 967 dollars. What are his total earnings? His average earnings per job?

After completing the work, add and sign the following statement:
I hereby certify that the above work is entirely my own.
(Signed)

Malcolm C. Sargent.

IMPORTANT

Show all Additions, Subtractions, Multiplications, and Divisions in your solutions.

Write plainly and distinctly.

Leave a space after each solution.

See that all solutions are enclosed with your examination before mailing.

Checking will show if results are accurate.

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